ICAB – failure criteria for beams (CM66)

The software ICAB Force/CM calculates the following criteria:
Sc  tensile stress criterion (CM66, CB71)
Tc  shear stress/(0.65 S0), (CM66)
Mc  Mises criterion (or Tsai-Wu)
F_cm66 buckling criterion (CM66 ou CB71)
D_cm66 buckling and warping criterion (CM66)
V_cm66 I-web warping criterion (CM66)

Definition of allowable stress

The standards used to define allowable stresses in members are REGLES CM 66 (AFNOR P 22-701) and CB71 (AFNOR P21-701).

Stresses are computed from resultant stresses (axial Nx, shear Ty, Tz, torsion Tx, bending My, Mz) with using the following formulas:

\[ \sigma_{\text{Nx}} = \frac{N_x}{A}, \quad \sigma_{\text{fy}} = \frac{M_y}{I_y}, \quad \sigma_{\text{fz}} = \frac{M_z}{I_z} \]

\[ \tau_{\text{xy}} = \frac{M_x}{J_{xy}}, \quad \tau_{\text{yz}} = \frac{T_y}{A_y}, \quad \tau_{\text{z}} = \frac{T_z}{A_z} \]

These stresses define the upper bounds of stresses \( \sigma_{xx}, \tau_{xy} \) et \( \tau_{xz} \).

\[ M_y = M_z = 0 \Rightarrow \sigma_{xx} = \sigma_{\text{Nx}} \]
\[ N_x = 0, M_z = 0 \Rightarrow |\sigma_{xx}| \leq |\sigma_{\text{fy}}| \]
\[ N_x = 0, M_y = 0 \Rightarrow |\sigma_{xx}| \leq |\sigma_{\text{fz}}| \]
\[ T_y = T_z = 0 \Rightarrow \tau_{xy}^2 + \tau_{xz}^2 \leq \tau_{\text{xy}}^2 \]
\[ T_z = 0, M_x = 0 \Rightarrow |\tau_{xy}| \leq |\tau_{xy}| |t_z| \]
\[ T_y = 0, M_x = 0 \Rightarrow |\tau_{xz}| \leq |\tau_{z}| \]

For a beam such as rectangular sections, the axial stress and shear are lesser than:

\[ |\sigma_{xx}| = \sigma \leq |\sigma_{\text{Nx}}| + |\sigma_{\text{fy}}| + |\sigma_{\text{fz}}| \]
\[ \tau_{xy}^2 + \tau_{xz}^2 \leq \tau_{\text{xy}}^2 + \tau_{z}^2 \]
\[ \tau_{xy} = |\tau_{xy}| + \max(|\tau_{xy}|,|\tau_{z}|) \]
\[ \tau_{z} = \min(|\tau_{xy}|,|\tau_{z}|) \]

For rounded beam sections:

\[ |\sigma_{xx}| = \sigma \leq |\sigma_{\text{Nx}}| + \sqrt{\sigma_{\text{fy}}^2 + \sigma_{\text{fz}}^2} \]
\[ \tau \leq |\tau_{xy}| + \sqrt{\tau_{xy}^2 + \tau_{z}^2} \]
2.1 traction (Sc)

Safety criterion is:

$$\frac{\sigma}{\sigma_e} < 1$$

Where \(\sigma\) is the maximum axial stress \(\text{(CM66 1,312)}\), i.e. \(\sigma\) is the sum \(\sigma_{Nx}+\sigma_{fy}+\sigma_{ff}\).

2.2 Shear (Tc)

$$\frac{\tau}{(0.65 \sigma_e)} < 1$$

Where \(\tau\) is the maximum shear stress \(\text{(CM66 1,313)}\), lesser than \(\tau^2 = \tau_1^2 + \tau_2^2\).

2.3 Von Mises criterion (Mc)

The Von Mises criterion (noted Mc) is used to check whether an isotropic material is yielding. For a symmetrical stress tensor, we have:

$$\frac{(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{xx})^2 + 6(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2)}{2\sigma_e^2} \leq 1$$

$$\Rightarrow \frac{\sigma_{xx}^2 + \sigma_{yx}^2 + \sigma_{zz}^2 - \sigma_{xx}\sigma_{zz} - \sigma_{yy}\sigma_{xx} - \sigma_{zz}\sigma_{yy} + 3(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2)}{\sigma_e^2} \leq 1$$

For a beam, we obtain:

$$\frac{\sigma_{xx}^2 + 3(\tau_{xy}^2 + \tau_{xz}^2)}{\sigma_e^2} \leq \frac{\sigma_e^2 + 3\tau^2}{\sigma_e^2} \leq 1$$

For a pipe under pressure, i.e. with a membrane stress \(\sigma_{in}\), the Von Mises criterion is:

$$\frac{\sigma_{in} + \sigma_{fy} - \sigma_{ff}}{\sigma_e^2} \leq 1 \quad \text{avec} \quad \sigma_{xx} = \sigma_{in} \pm \sigma_f$$

with:

- \(\sigma_{Nx}\) tensile stress related to axial force
- \(\sigma_t\) stress related to bending moments

NB. The Von Mises criterion reported by ICAB is the square root of the preceding formula so that this criterion is proportional to applied load.

2.4 Buckling (F_cm66)

For a beam submitted to flexion and bending, the section 3,521 of CM66 (noted F_cm66) indicates that we must have:

$$\frac{k_1\sigma_{Nx} + k_f\sigma_{fy} + k_f\sigma_{ff}}{\sigma_e} \leq 1$$

The coefficient \(k_1\) corresponds to the buckling coefficient for exceptional checking (CM66 3,412).

$$k_1 = \frac{\mu_i - 1}{\mu_i - 1,3}, \quad \mu_i = \frac{\sigma_k}{\sigma_{Nx}}$$

$$\sigma_k = \frac{\pi^2 E}{\lambda^2}, \quad \lambda = \max(\lambda_y, \lambda_z)$$

$$\left(I_{yz} \neq 0 \Rightarrow \lambda = \lambda_y\right)$$
The coefficient $k_y$ for the amplification of bending stresses is the most unfavorable corresponding to a constant bending moment of with a linear variation (CM66 3.513):

$$k_{fy} = \frac{\mu_y + 0.25}{\mu_y - 1.3}, \quad \mu_y = \frac{\sigma_{ky}}{\sigma_{Ny}},$$

$$\sigma_{ky} = \frac{\pi^2 E}{\lambda_y}, \quad \lambda_y = \frac{l_{ky}}{I_{xy}} \sqrt{\frac{1}{A}}$$

Similar calculations are conducted for compression and bending $M_z$.

2.5 Warping ($D_{cm66}$)

The envelope formula CM66 3.731 (noted $D_{cm66}$) is used:

$$\frac{\sigma_{Ny} k + \sigma_{fy} k_{dy} \lambda_{dy} + \sigma_{fy} k_{de} \lambda_{de}}{\sigma_e} \leq 1$$

The stress $\sigma_{Ny}$ is taken into account if the beam is subject to compression.

The buckling coefficient $k$ is (CM66 3.411):

$$k = (0.5 + 0.65 \frac{\sigma_e}{\sigma_h}) + \sqrt{\left(0.5 + 0.65 \frac{\sigma_e}{\sigma_h}\right)^2 - \frac{\sigma_e}{\sigma_h}}$$

$$\sigma_k = \frac{\pi^2 E}{\lambda^2}, \quad \lambda = \max(\lambda_y, \lambda_z)$$

$$l_{xz} \neq 0 \Rightarrow \lambda = \lambda_z$$

The warping coefficient $K_{dy}$ is (CM66 3.611):

$$\sigma_{dy} \geq \sigma_e \Rightarrow k_{dy} = 1$$

$$\sigma_{dy} < \sigma_e \Rightarrow k_{dy} = \frac{k_0}{1 + \frac{\sigma_{dy}}{\sigma_e} (k_0 - 1)}$$

$$\sigma_{dy} = \frac{\pi^2 E}{5.2} \frac{l_{zz} h_z^2}{I_{yy} l_{dy}} (D - 1) BC$$

$$k_0 = \left(0.5 + 0.65 \frac{\sigma_e}{\sigma_{x0}}\right) + \sqrt{\left(0.5 + 0.65 \frac{\sigma_e}{\sigma_{x0}}\right)^2 - \frac{\sigma_e}{\sigma_{x0}}}$$

$$\sigma_{x0} = \frac{\pi^2 E}{\lambda_{x0}}, \quad \lambda_{x0} = \frac{l_{Dy}}{h_z} \sqrt{4 \frac{l_{yy} I_{zz}}{BC I_{zz}} \left(1 - \frac{\sigma_{Dy}}{\sigma_e}\right)}$$

The coefficients B, C, D are used to take into account the distribution of loads and dimensions of the section:

Reference www.icab.fr v4
\[ D = \sqrt{1 + \frac{4}{\pi^2} \frac{GJ}{E I} \frac{l_k^2}{h_z^2}} \]

\[
C = \sqrt{\frac{3}{1 + \frac{M_e}{M_w} + \left( \frac{M_e}{M_w} \right)^2 - 0.152 \left( 1 - \frac{M_e}{M_w} \right)^2}}
\]

\[
B = \sqrt{1 + \left( \frac{z_a 8 \beta C}{h_z \pi^2 D} \right)^2 \frac{z_a 8 \beta C}{h_z \pi^2 D}}
\]

The calculation of C is truly valid for beams subjected to bending moments at the supporting points with flanges free to rotate around local beam axis (z); M_w and M_e are these two bending moments, M_w being the highest. The coefficient B used for the calculation is the smallest of B values calculated at \( z_a = 0, h/2 \). The coefficient b=3 corresponds to a uniform load distributed on a beam free to rotate around its extremities around local axis (z) (CM66 3,643).

### 2.6 Buckling lengths

The calculations of failure criteria use 4 lengths:

- \( l_{ky}, l_{kz} \): buckling lengths in the two local planes XoZ and XoY of the beam
- \( l_{Dy}, l_{Dz} \): warping lengths in the two local planes XoZ and XoY of the beam

Hence, the length \( l_{ky} \) is used for the buckling in the XoZ plane that may occur with stress resultant \( N_x \) (compression) and \( M_y \) (moment around y axis).

In this note, these lengths are at least equal to the length of the finite element beam, i.e. distance between nodes. These lengths may be increased by specifications defined in physical properties of the beam:

- \( LKY \): minimum buckling length
- \( LKZ \): minimum buckling length
- \( LDY \): minimum warping length
- \( LDY \): minimum warping length

In all members used in the model, the buckling and warping lengths are at least equal to the length of the beam between nodes.

For horizontal bracing, the out-of-plane buckling length is the distance between legs.

NB. The program used for the present simulations computes the criteria \( F_{cm66} \) (buckling) and \( D_{cm66} \) (buckling and warping) for all sections, even for hollow sections, for which the warping failure may be not relevant.